Abstract - A novel multi-camera corneal topographer arrangement is proposed in the paper. The arrangement uses a novel colour-coded measurement pattern that facilitates correspondence detection. The topographer relies on a novel surface reconstruction method. It takes images shot by up to four synchronized cameras as inputs. The surface reconstruction is accomplished by the joint solution of multiple partial differential equations (PDE's). Each of these PDE’s describes the phenomenon of light reflection on specular corneal surface with respect to the common measurement pattern on one hand, and a particular camera in the multi-camera arrangement on the other. To some extent, even the present version of the topographer arrangement makes use of embedded computing concepts; however, their presence and importance in the design are expected to increase with the newer versions of the measurement device.

I. INTRODUCTION

The cornea is the primary optical structure that focuses the light entering the eye. Other structures in the light-path – including also the crystalline lens – have smaller refractive power. The coating of the cornea's outer surface is called pre-corneal tear film. This tear film is replenished with every blink, thus providing after its build-up a smooth optical surface on the microscopically irregular corneal surface.

Due to the high refractive power of the cornea, the knowledge of its detailed topography is of great diagnostic importance. Examination devices, such as keratometers, corneal topographers, and examination methods used in ophthalmology for exploring and measuring these topographies have a relatively long history. See the paper of Jongsma et al. [4] for the bibliographical details of papers written by Helmholtz, Placido and Gullstrand in the 19th century.

Nevertheless, the explosive growth in numbers, as well as, in precision and in professional acceptance of such devices and methods came along with the availability of inexpensive computing power incorporated in the PC's, that is, in the early 1990's. The recent advancement of embedded computing facilities and devices may well signal a further boom of such devices.

Nowadays, the corneal topographers are used in a wide range of ophthalmic examinations. They are used in the diagnostics of corneal diseases, in contact lens selection and fitting, in planning sight-correcting refractive surgical operations, and in their post-operative check-ups just to mention a few [5]. Also, dynamic properties – e.g., the average build-up time – of the pre-corneal tear-film can be examined and measured using fast-operation corneal topographers [7]. In Section II., the measurement anomalies experienced with existing reflection-based corneal topographers are outlined.

In Section III, the most important advances in the field of specular surface reconstruction are summarized. In Section IV, a novel reflection-based corneal topographer arrangement that eliminates most of the aforementioned measurement anomalies is proposed.

In Section V, the embedded aspects of the envisaged corneal topographer system – based on the topographer arrangement proposed herein – are described. Section VI concludes the paper and outlines the work that is necessary to reach a more mature stage in the product life cycle.

II. CORNEAL MEASUREMENTS ANOMALIES EXPERIENCED WITH KNOWN REFLECTION-BASED TOPOGRAPHERS

The majority of the measurement methods applied in the presently used corneal topographers relies on the specularity of the pre-corneal tear film that is coating the otherwise non-specular corneal surface. In most of topographers, some bright measurement pattern of known and well-defined geometry, e.g., a concentric system of bright and dark rings, called Placido-rings, placed on a
planar, spherical, conic or cylindrical surface is generated and displayed in front of the eye, so that the reflection of the measurement pattern on the pre-corneal tear film can be photographed by a camera. The photograph should be taken when the relative position of the corneal surface – with respect to the measurement pattern – has been properly adjusted by the ophthalmologist.

Then the measurement pattern’s reduced and somewhat distorted virtual image (Fig. 1.) is recorded and quantitatively analyzed, so that examined corneal surface can be mathematically reconstructed. Based on this reconstruction, maps showing the topography of corneal surface and its local optical properties are produced and displayed, furthermore, its various optical aberration features, such as Zernike coefficients, are calculated and displayed for the ophthalmologist.

In case of healthy and regular corneal surfaces, the presently available corneal topographers generally produce good quality corneal snapshots, and based on these they produce precise and reliable optical power maps. However, even for healthy and regular surfaces, an impurity of microscopic size in the pre-corneal tear film can produce a significant measurement anomaly.

Simple measurement patterns, such as the Placido ring systems that are widely used in presently available corneal topographers, do not provide the necessary information to correct a local measurement anomaly. In such a case, ring-points could be easily missed by the ring-detection program. Such a measurement anomaly is shown in Figs. 2 and 3.

Missed ring-points result in rings with high curvature-variance near these misses. Such rings in turn result in significant and extensive, i.e., non-local errors in the generated optical power map. The optical power map for the Placido rings detected with ring-point misses that has been shown in Fig. 2 is presented in Fig. 3.

The dynamic behaviour of the pre-corneal tear film coating the corneal surface keeps changing between blinks. In the first few seconds after a blink, the tear film spreads over and builds up on the corneal surface.

Then it begins to evaporate and after a while it gradually breaks up. The optical smoothness of the corneal surface is guaranteed only during the evaporation phase of the pre-corneal tear film. Snapshots taken at other times exhibit various degrees of measurement anomaly. Another type of tear film related measurement anomaly is linked to the intense watering of the eye. In this case, the thick tear layer near the lower eye-lid falsifies the measurement results.

Weird and incorrect optical power maps are produced, if the measurement pattern has not been properly centered onto the cornea by the ophthalmologist carrying out the examination, or if the image taken was blurred because of the improper camera-eye distance setting used.

The measurement anomalies are even greater, if callused, irregular corneal surfaces are examined using a Placido-based topographer [13]. In such a case, many of the reflected ring-points are indistinguishable and it is difficult, if at all possible, to determine the original radial order of the ring-segments; see Fig. 4 for such a problematic case. Consequently, only a partial, rough and very unstable surface reconstruction can be achieved that is of no use for any diagnostic purposes.
III. ADVANCES IN THE RECONSTRUCTION OF SPECULAR SURFACES

The mathematical reconstruction of specular surfaces has been an active area of research in computer vision recently. Many important papers were published in this area, e.g., [2], [3], [9] to cite a few.

A. Local reconstruction of specular surfaces

For a given pair of an object-, and an image point, there are usually an infinitely great number of specular surface-patches that could cause a light-ray starting from the object-point to be reflected in a manner that it reaches the image-point. In order to find out which of these patches is the right one – i.e., the one that really exists and is responsible for the reflection – it is necessary to gain further information. This information could concern the global shape of the specular surface. The general conditions for and the limitations of the local reconstruction of specular surfaces (i.e., specular surface patches) are dealt with in great detail in [8], [11].

In case of reconstruction of planar specular surface, in addition to the mentioned object-point–image-point correspondence, one must use a further correspondence for the reconstruction of the planar surface. This could be the correspondence of straight lines due to reflection: one straight line passing through the object-point and the other passing through image-point. With this information, the plane of the planar specular surface can be obtained uniquely. Another relatively simple case is that of the reconstruction of spherical specular surface. In this case, one must rely on an additional correspondence (i.e., constraint) in order to achieve a unique local surface reconstruction. This constraint, for example, could be a second pair of corresponding straight lines – more precisely tangents of smooth curves – intersecting the first pair of straight lines in the relevant object-, or image-point.

For the reconstruction of a general smooth surface-patch – such a patch could be approximated with a second-order surface near the reflection point, and therefore could be described with two curvature and an angular data – three corresponding straight (tangent) line pairs intersecting in the object- and image-point, respectively, could be considered. However, it is shown in [11], that this information is not sufficient for unique local surface reconstruction in general. A unique reconstruction can be achieved though, if also the local curvatures of the intersecting curves are considered. Savarese and his co-authors note that their results are valid only for exact measurements. Effects of various noise factors were not considered in the referred papers. This limitation may suggest that the local surface reconstructions are only the initial steps forward in the process of a global reconstruction of the unknown specular surface.

To achieve a global surface reconstruction, it seems to be important to rely on the smoothness of the surface, and to examine (i.e., look at) the surface-patches from one or more other viewpoint and use this information for the reconstruction.

B. Global reconstruction of specular surfaces

The reconstruction of a specular surface from several views cannot be achieved via the popular algorithms of stereo vision [9]. This is because, these algorithms rely on segments, points, curve-segments, textures, colours, etc. – that appear and can be identified in several images – being fixed in 3D space. In case of specular surfaces, this property of segments, points, curve-segments simply does not exist. For this reason, the specular reflections appearing on multi-view images hinder, rather than help, the reconstruction of the scene when the mentioned algorithms are used. Consequently, other approaches and algorithms are required for the reconstruction of specular surfaces from several views. Several interesting papers were published on this topic recently e.g., [9], [10], [12].

The normal of a given specular surface-patch is the same no matter which camera of a multi-camera arrangement looks at it. Trivially, the normal itself cannot be seen, it can be examined and calculated indirectly by considering its effect with respect to the reflection it causes to various light-rays. By doing this for a great number of patches, information can be gathered about the normals of an extensive specular surface.

Intuitively, those points are located on, or in the vicinity of a specular surface for which the corresponding unit normals calculated from two or more views are approximately the same.

This observation serves as the basis of the voxel-carving method suggested by [9]. This method can be used for surface-points – and spatial points in the vicinity of the surface-points – that produce identifiable reflections of the measurement pattern in more than one view.

A theoretically more profound approach was proposed in [10]. In their approach, the description of light-reflection by a smooth specular surface takes the form of a total differential equation.

IV. THE PROPOSED MULTI-CAMERA CORNEAL TOPOGRAPHER ARRANGEMENT

The proposed corneal topographer arrangement comprises a measurement pattern generator and up to four
cameras. A three-camera arrangement is shown in Fig. 5; and a camera view of a patient’s cornea is shown in Fig. 6.

A. Camera calibration

When using cameras for 3D measurement purposes, the camera calibration is an essential task. There are several camera calibration algorithms and programs that are widely used worldwide for calculating the internal and external camera parameters, e.g., [6], [14].

For the calibration of the cameras in the topographer arrangements shown in Figs. 5 and 6, a new Matlab toolbox was developed. Its calibration approach is very similar to that of the above cited calibration methods, but it is more flexible in use, and more importantly, facilitates the selection and the use of a set of practical – and in our case realistic – restrictions to be considered during parameter optimisation.

These include “no skew”, “square pixels”, “central position of the principal point”, “no distortion” restrictions. The use of these restrictions ensures that a realistic calibration is reached with reasonably fast convergence.

In Fig. 7, a chessboard pattern is placed on a wedge for calibration purposes. The wedge was turned by a horizontal turntable around its vertical axis to fixed angular positions with selectable angular increments of 10, 15, 20, 30, 45, 60 and 90 degrees.

In Fig. 8, a tableau of 12 images shows the mentioned chessboard pattern in rotated positions. The selected angular increment in this case was 10 degrees. The images of the tableau show also the selected origins and coordinate axes.

The goodness of the calibration method is demonstrated by the reconstructed positions of chessboards shown in Fig. 9. To maximize the demonstrative power of figure, special care was taken to select the origin and the coordinate axes in a consistent manner throughout the image sequence.

This clearly indicates the usability of this calibration method and toolbox in the given context and for the cameras in the proposed multi-camera corneal topographer arrangement.
B. Selection of the measurement pattern

It has been pointed out in Section II, that a more elaborate and more informative measurement pattern is necessary for robust corneal measurements than the frequently used Placido ring-system. In the known reflection-based corneal topographers a wide range of measurement patterns are used. This range includes various black and white and colour-coded Placido ring-systems (Fig. 10a) in planar, spherical, paraboloid arrangement, the AstraMax corneal topographer’s radial chessboard-like pattern, and on the more complex side, various colour-coded patterns (Fig. 10b), such as suggested in [15].

For the proposed topographer arrangement, a four-coloured (e.g., red, green, blue and yellow) rectangular array-structure was used as measurement pattern. The pattern has unique 3-by-3 field-neighbourhoods.

This uniqueness of neighborhoods is preserved even in rotated and distorted images. A colour-code with unique 4+1 neighbourhoods was suggested in [1] for a similar purpose. However, there only four rotations (i.e., $k \times 90$ degrees) are considered in the generation and the checking of the colour-code, while in the proposed colour-code eight rotations (i.e., $k \times 45$ degrees) are considered.

For the proposed topographer arrangement, a four-coloured (e.g., red, green, blue and yellow) rectangular array-structure was used as measurement pattern. The pattern has unique 3-by-3 field-neighbourhoods.

C. Algorithm for reflection-based single-view corneal surface reconstruction

Mathematically, the tear-film coated corneal surface is modelled with a smooth, convex surface $F$. This surface is described – and sought – in a preferably chosen spatial polar-coordinate system; namely in one that has its origin $B$ in the camera's optical centre – see the geometrical arrangement and the notation used in Fig. 13 – and its axis is the optical axis $BB'$ of the camera.

The surface $F$ is described in the following form:

$$F(x_1, x_2) = S(x_1, x_2)x'$$
where $x^* = (x_1, x_2, 1)^T$. Here, $S(x)$ ($x = (x_1, x_2)^T$) is the distance – measured from $B$ – of the intersection point $P$ defined by the light ray emitted from $B$ in direction $X = P_B B$ on one hand, and the specular surface $F$ on the other. The propagation of light from the points of the measurement pattern to the reflected and possibly distorted image taken by the camera – e.g., $PxBB$ – is described in the mentioned polar coordinate system.

By doing so, a mapping is identified between the points $P_{y}$ of the measurement pattern and the points $P_{x}$ of the image. It follows from the conditions prescribed for the surface that this mapping is one-to-one. Such a mapping is illustrated in Fig. 14 for a particular measurement setup.

As follows from the physical law of light-reflection, the two-variable function $S(x)$ describing surface $F$ satisfies the following first-order partial differential equation:

$$\frac{1}{S(x)} \frac{\partial}{\partial x_j} S(x) = \frac{(v_j(x) - x_j)}{|x^* \cdot (x^* - v(x))|}$$

where

$$v(x) = |x^*| \left(\frac{k + f(x) - S(x) x^*}{|k + f(x) - S(x) x^*|}\right),$$

and function $f(x)$ can be expressed with the inverse of the mentioned $P_y \rightarrow P_x$ mapping, that is, with mapping $P_x \rightarrow P_y$.

Referring to Fig. 13, $f(x) = KP_y$ where $K$ is the origin of the coordinate system chosen in the plane of the measurement pattern, while $k = OK$ denotes a vector pointing to point $K$. It follows from the mathematical model described above that surface $F$ can be determined uniquely under the starting condition $S(0, 0) = s_0$, if the $P_y \rightarrow P_x$ mapping is known.

A numerical procedure – taking discrete values of the mapping $f(x)$ as input – has been devised, firstly, to calculate the $P_x \rightarrow P_y$ mapping, and secondly, to solve the mentioned partial differential equation. In Fig. 15, a surface – together with its normals – reconstructed from a single camera-view using the mentioned numerical procedure is shown for the artificial test cornea.

In simulations carried out for known surfaces, good approximations of the original surfaces and their various curvatures were produced via the mentioned numerical surface reconstruction procedure.

**D. Algorithm for reflection-based multi-view corneal surface reconstruction**

In case of a multi-camera arrangement, the solution of the aforementioned PDE must start from a surface-point reflecting the measurement pattern to at least two cameras.

Let $C_i$ denote the image of the reflected measurement pattern taken by $i$-th camera, and $F_i$ the part of the corneal surface actually reflecting the measurement pattern into the $i$-th camera.

From these simulations it has turned out that the surface reconstruction procedure is clearly sensitive to the starting condition $s_0$, while it is much less sensitive to errors present in the $P_y \rightarrow P_x$ mapping.

The $F_i$ and $F_j$ surface-regions corresponding to the $i$-th and the $j$-th cameras of the proposed arrangement usually have overlapping regions. An algorithm has been devised that determines the distances of an arbitrarily chosen point of the overlapping surface-region from the $i$-th and the $j$-th cameras based on $C_i$ and $C_j$ images.

This point – and these distances – will serve as the starting condition for the $i$-th and the $j$-th PDE corresponding to the $i$-th and $j$-th cameras, respectively).

After appropriate fitting, the union of the surface-regions will provide the reconstructed surface. Unit normal vectors, and the various curvatures used by the ophthalmologists can be calculated for any surface points.
V. THE EMBEDDED ASPECTS OF THE TOPOGRAPHER BASED ON THE PROPOSED ARRANGEMENT

The requirements concerning a computer that needs to be embedded in some electronic system are usually manifold. The optimal choice of the embedded computer, its peripherals and operating system is influenced by a wide range of technical and non-technical factors. Requirements concerning the small physical size, low weight and the low power-consumption are quite common.

Furthermore, the embedded computer should be able to fulfill the necessary control and communication tasks arising in the host system, between the embedded computer and the host system, etc. In many cases, the embedded computer is required to provide significant computing power.

The block diagram of the proposed corneal topographer's embedded implementation is given in Fig. 16. From the diagram, it is clear that several peripherals – including cameras – are to be used in the system.

This implementation will use a commercial industrial PC base-board that was explicitly designed for embedded systems. Such embedded PC-boards are capable of running, and usually provided with popular operating systems (e.g., MS Windows XP, MS Windows XPE, certain Linux-version).

The requirements concerning a computer that needs to be embedded in some electronic system are usually manifold. The optimal choice of the embedded computer, its peripherals and operating system is influenced by a wide range of technical and non-technical factors. Requirements concerning the small physical size, low weight and the low power-consumption are quite common.

One of the main advantages of using such operating systems is that peripherals of the embedded system can be chosen from a huge set and their programming and handling are fairly straightforward in the mentioned operating systems.

The embedded platform selected according to considerations outlined above per se satisfies the requirements concerning the experimental system. At the same time, provide an up-to-date, efficient and reasonably low-cost solution for the product-level corneal topographer that will be based on the developed prototype. The incorporation of other embedded platforms – such as RISC-processors, DSP's – in the final product may well have benefits, but their obvious drawback of increased developmental time and costs was prohibitive in the given case.

VI. CONCLUSION

The most existing corneal topographers provide satisfactory precision and generate realistic optical power maps in case of healthy, regular corneas. For irregular surfaces, the simplistic measurement patterns, such as the Placido-ring system, do not provide sufficient information to properly reconstruct the corneal surface.

Majority of the topographers in use, rely on one view only, which is theoretically – and in case of irregular surfaces also practically – insufficient for the unique reconstruction of the corneal surface. To overcome this essential measurement deficiency several – up to four – cameras were included in the corneal topographer arrangement.

Up-to-date embedded computing approaches and devices are to be used in the embedded version of the proposed topographer arrangement. The embedded concepts used in the present prototype and those to be used in its embedded version are suitable for the product stage, as well.

Test measurements are presently being carried out on living corneas. The measurement results gained in these test measurements will be compared to measurement results gained from other devices.

ACKNOWLEDGMENT

This research has been partially supported by the National Office for Research and Technology, Hungary, under NKFP-2/020/04 research contract.

REFERENCES


