Nonlinear Digital Signal Processing

Nonlinear filters

- **Order statistic filters**
- **Homomorphic filters**
- **Polynomial filters**
- **Implementation of nonlinear filters**

Neural networks

- **Static networks**
- **Dynamic (temporal) networks**
- **Implementation of neural networks**

Nonlinear Filters

Order statistic filters

Median filters

$$med(x_i) = \begin{cases} x_{\nu+1} & n = 2\nu + 1 \\ x_\nu + x_{\nu+1} & n = 2\nu \end{cases}$$

where

$$x_1 \le x_2 \le \cdots x_\nu \le x_{\nu+1} \le \cdots \le x_n$$

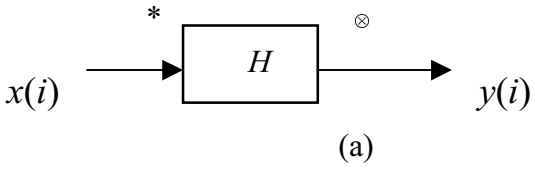
1-D case

$$y_i = med\,(x_{i-\nu}, \ldots x_i, \ldots x_{i+\nu}) \qquad i \in \mathbf{Z}$$

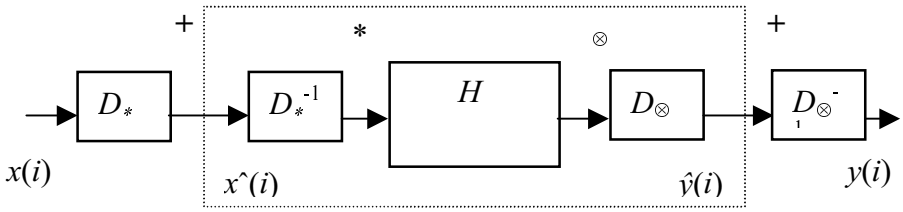
2-D case

$$y_{ij} = med\left\{x_{i+r,\,j+s}\,;\ \ (r,s) \in A\right\} \qquad (i,j) \in \mathbf{Z}^2$$

Homomorphic filters



(a)



(b)

$$\hat{x}(i) = D_*[x_1(i)] + D_*[x_2(i)] = \hat{x}_1(i) + \hat{x}_2(i)$$

$$\hat{y}(i) = L[\hat{x}(i)] = L[\hat{x}_1(i) + \hat{x}_2(i)] = \hat{y}_1(i) + \hat{y}_2(i)$$

$$y(i) = D_{\otimes}^{-1}[\hat{y}_1(i) + \hat{y}_2(i)] = y_1(i) \otimes y_2(i)$$

$$H[x_1(i)*x_2(i)]=H[x_1(i)] \otimes H[x_2(i)]$$

Polynomial filters

$$y(n) = h_0 + \sum_{k=1}^{\infty} \overline{h}_k[x(n)]$$

$$\overline{h}_k[x(n)] = \sum_{i_1=0}^{N-1} \ldots \sum_{i_k=0}^{N-1} h_k(i_1,\ldots,i_k)x(n-i_1)\ldots x(n-i_k)$$

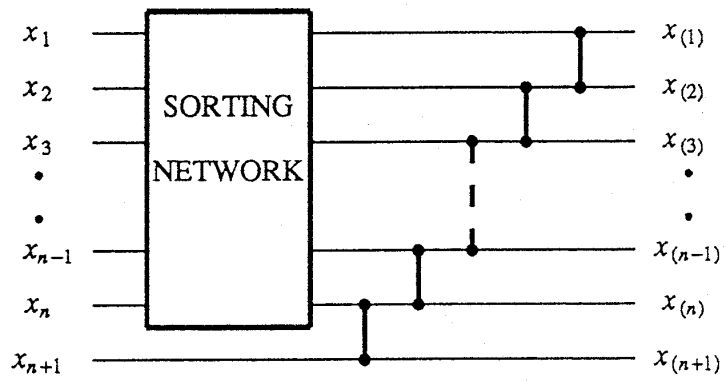
Quadratic filters

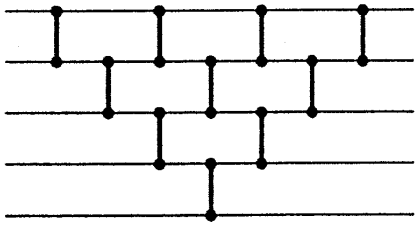
$$y(n) = \sum_{i_1=0}^{N-1} \sum_{i_2=0}^{N-1} h_2(i_1,i_2)x(n-i_1)x(n-i_2)$$

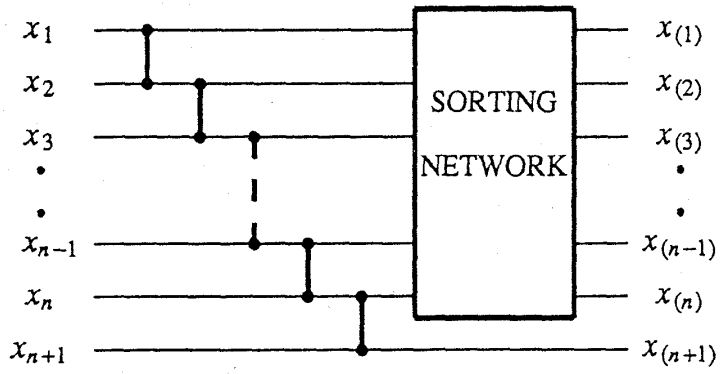
$$y(n) = \sum_{i=1}^{N} \sum_{j=1}^{N} h_{i,j} x_{n-i+1} x_{n-j+1}$$

Implementation of nonlinear filters

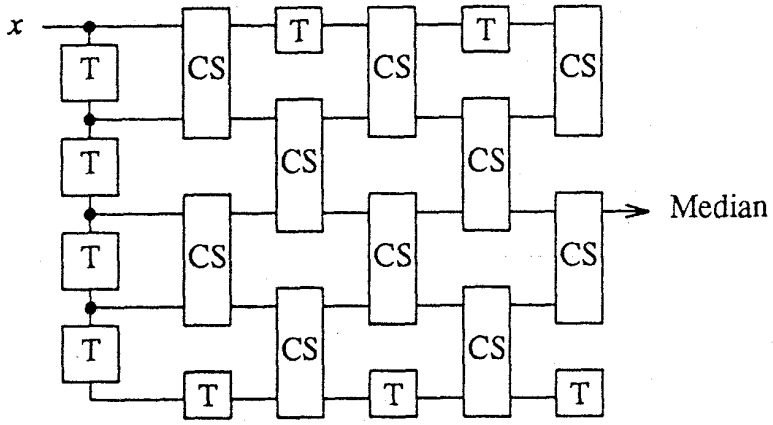
Sorting

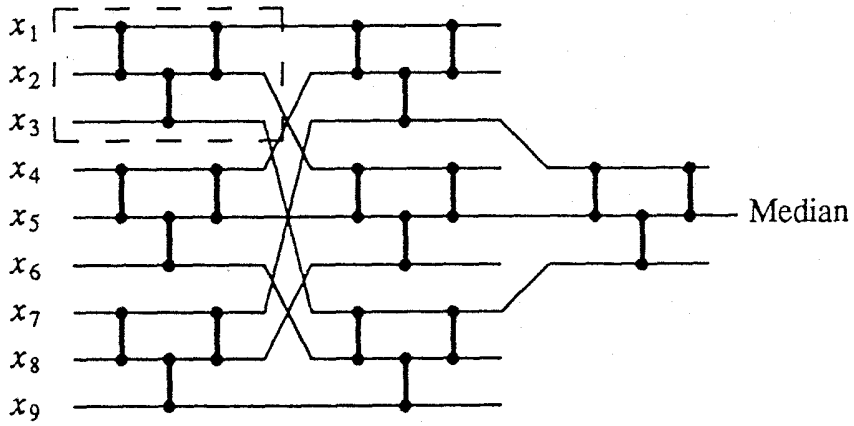




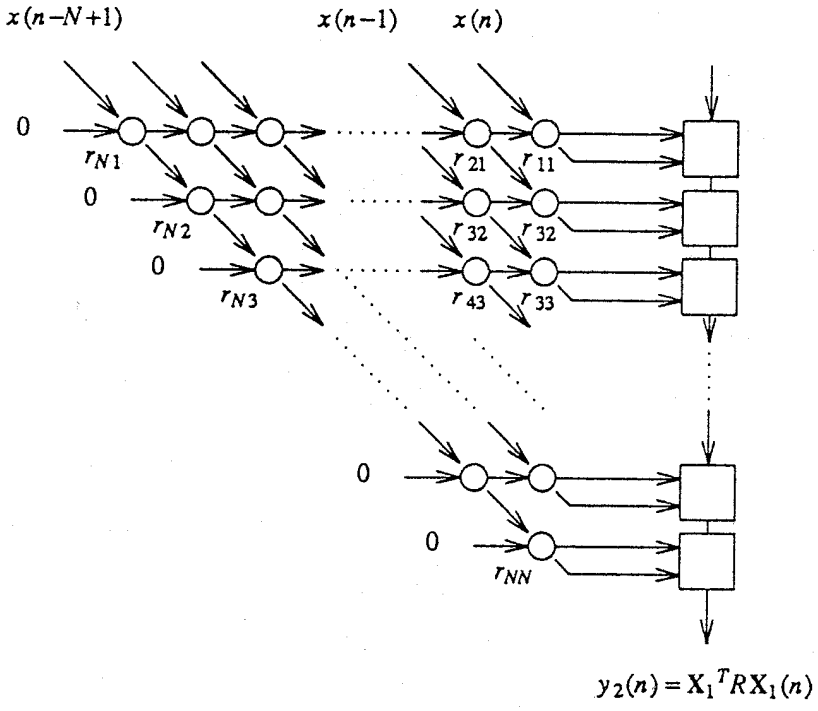


Structures for median filtering

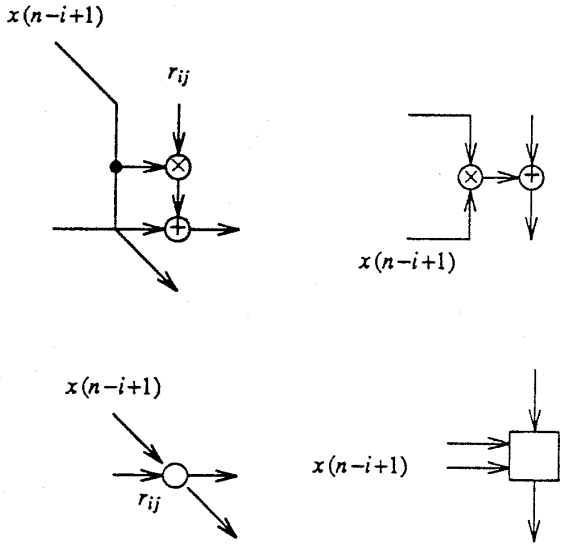
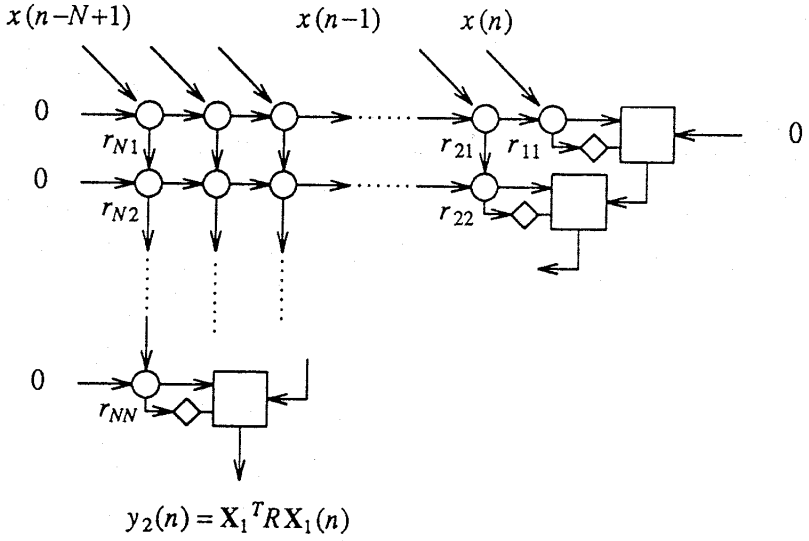




Systolic array for quadratic filters



(a)



Wavefront array of quadratic filter